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On viscous holographic dark energy universe with Nojiri-Odintsov cut-off

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Abstract

In this paper we would like to report about recent research related to the accelerated expansion of large scale universe due to a specific form of a holographic dark energy model with Nojiri-Odintsov cut-off including a specific form of parametrization of viscosity. Results presented here have been discussed during QFTG 2016 and SCD 2016 at Tomsk.

keywords: accelerated expansion, the large-scale universe, dark energy.

1 Introduction

The accelerated expansion of large scale universe is one of long standing problems of physics puzzling researchers [1] - [3]. A short review of literature on problems of modern cosmology shows that mainly three directions were developed to solve it (and related problems). In general, according to recent understanding of physics of large scale universe DE ($\approx 70\%$) provides acceleration to expanding universe; it takes the properties to be mysterious with negative pressure working against gravity [4] (and references therein). In modern cosmology, cosmological constant is the simplest model of DE ever suggested. However, mainly two problems arise with that model discussed in recent literature very intensively [5] (and references therein). In particular, fine - tuning problem indicating absence of a fundamental mechanism, which sets cosmological constant to zero or to a very small value, because in the framework of quantum field theory, the expectation value of the vacuum energy is 123 order of magnitude larger than the observed value.

One of the first approaches for solving the mentioned problems is based on dynamical DE models. One of the first models of varying DE considered in literature is the varying cosmological constant having mainly a phenomenological origin [6], [7] (and references therein). Among DE models are quintessence, phantom and quinton DE models, holographic and tachyonic DE models (see Ref. [4] and references therein). An active discussion is hold on the accelerated expansion of large scale universe involving dark fluids – Chaplygin gas (and its modifications), van der Waals gas and polytropic gas involving also viscosity, among others see [8] - [24] and references therein. On the other hand, various modifications of the field equations on the lagrangian level have been considered as more fundamental giving a wide range of modified GR theories [25], [26] (and references therein). Consideration of modified GR promises new insight into understanding of universe (and it is one of the hot topics for study). Due to the fact, that all approaches aim to explain available observational data and the constraints on the models are imposed by them, it is necessary to have appropriate tools to distinguish suggested models from each other [27] - [31]. Recently, in Ref [32] an attempt to explore nature of accelerated expansion of large scale universe has been taken. Model considered in Ref. [32] involves a generalized holographic DE model with a Nojiri - Odintsov cut - off and pressureless CDM. For the first time the generalized holographic DE with the Nojiri-Odintsov cut - off defined as

$$\rho_{DE} = \frac{3c^2}{L^2}, \quad (1)$$

with

$$\frac{c}{L} = \frac{1}{L_f} [\alpha_0 + \alpha_1 L_f + \alpha_2 L_f^2] \quad (2)$$

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where L_f it is the future horizon and defined as

$$L_f = a \int_t^\infty \frac{dt}{a}, \quad (3)$$

while c , α_0 , α_1 and α_2 are numerical constants, has been considered in Ref [12]. Considered model of DE gives possibility of unifying of early - time and late - time universe based on phantom cosmology. Moreover, one of interesting results (among others) discussed in Ref. [12] is related to possibility of phantom - non-phantom transition, which appears in such a way that universe could have effectively phantom equation of state at early - time as well as at late - time. In this paper, we would like to consider different possibility replacing CDM with viscous dark fluid of a certain type (conditionally named DM)

$$P_{DM} = -\rho_{DM} + \rho_{DM}^\alpha + \chi H^\beta, \quad (4)$$

where α , χ and β are parameters to be determined from observations and giving positive energy density for the fluid. The dynamics of the background for considered toy model is assumed to be GR. Moreover, if there is non - gravitational dynamics between these components (which is known as interaction), then to describe the dynamics of energy densities we should have

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1 + \omega_{DE}) = -Q, \quad (5)$$

$$\dot{\rho}_{DM} + 3H\rho_{DM}(1 + \omega_{DM}) = Q, \quad (6)$$

where ω_{DE} is EoS parameter of DE and ω_{DM} is EoS parameter of DM. In particular, we will consider the following form of interaction

$$Q = 3Hb(\rho_{DE} + \rho_{DM}), \quad (7)$$

and compare results with a different model assuming the following form of sign changeable interaction

$$Q = 3Hbq(\rho_{DE} + \rho_{DM}), \quad (8)$$

where b is positive constants for both cases.

The paper is organized as follows: In section 2 we will present and discuss results from cosmographic analysis of models involving different forms of interaction. In section 3 we will study the models involving Om and two point Om analysis. Moreover, validity of generalized second law of thermodynamics is demonstrated. Finally, a discussion on obtained results and possible future extension of considered cosmological models are summarized in section 4.

2 Cosmography

To simplify discussion relevant with recent observational data, as in case of Ref. [32], the following constraints $\alpha_0 \in [0, 1]$, $\alpha_1 \in [0, 1]$ and $\alpha_2 \in [0, 1]$ on parameters of the generalized holographic Nojiri - Odintsov DE model are taken into account. Moreover, the best fit of theoretical results with distance modulus has been considered. In this case non - interacting model describes by the following deceleration parameter

$$q = \frac{-2\sqrt{\Omega_{de}}\dot{L}_f(\alpha_1 + 2\alpha_2 L_f) - H\Omega_{de}(\dot{L}_f + 1)}{2H^2 L_f} + \frac{L_f(H^2 + P_{DM})}{2H^2 L_f}. \quad (9)$$

Fig. (1) represents the graphical behavior of the deceleration parameter, Ω_{DE} , Ω_{DM} , ω_{DE} and ω_{DM} versus redshift for different values of viscosity parameter χ .

The top - left plot of Fig. (1) demonstrates decreasing nature of the deceleration parameter q indicating present day value increase of it with increase of χ . Moreover, observed increase of present day value of the deceleration parameter significantly affects on transition redshift z_{tr} , in particular, increase of χ brings appropriate decrease of transition redshift z_{tr} . The top - right plot of Fig. (1) demonstrates the graphical behavior of Ω_{DE} and Ω_{DM} parameters versus redshift z and that the model is free from cosmological coincidence problem. Interesting behavior has been observed for EoS parameter of DE. In particular, bottom - left plot demonstrates that during considered period of evolution $\omega_{DE} < 0$, moreover, decreasing and increasing nature of EoS parameter provides phantom crossing in near past giving present day value of ω_{DE} well comparable with recent observational data. In summary, behavior of ω_{DE} indicates transitions between either past phantom universe, or quintessence universe and recent quintessence universe. This is due to viscosity, which in its turn gives to ω_{DM} graphical behavior presented in bottom - right plot of Fig. (1). From this plot we see, that there is a specific range of the model parameters, when $\omega_{DM} \rightarrow 0$. This case is interesting since, in standard model of cosmology matter is assumed to be pressureless. The values of model parameters corresponding to this case are $\alpha = 1.15$, $\chi = 0.1$, $\beta = 1.2$, $\alpha_0 = 0.15$, $\alpha_1 = 0.2$, $\alpha_2 = 0.25$ with $H_0 = 0.7$ and $\Omega_{DM} = 0.27$. It can

be also seen that depends on the value of viscous parameter χ at low redshifts matter described by Eq. (4) can be either DE, or usual matter. On the other hand at higher redshift it is always matter with $\omega_{DM} > 0$. During our study we also found another interesting behavior of the matter given by Eq. (4) which can be interesting for another research. In particular, with $\alpha = 0.85$ and $\chi = -0.1$ $\omega_{DM} \rightarrow 0$ can be observed, however, in this case during considered redshifts of evolution it will be a fluid with $\omega_{DM} < 0$ only. Presented, both possibilities are interesting for further study, in particular, it will be interesting to study structure formation problem and understand about impact of the viscosity on the process corresponding to mentioned regimes. Comparison of the behavior of the deceleration parameter q and ω_{DE} for various values of α parameters for $\chi = -0.1$ and $\chi = 0.1$ showed that for lower values of α only ever accelerated expanding universe will be observed. On the other hand, transition universe will be observed with increasing of α giving decreasing transition redshift z_{tr} with decreasing present day value deceleration parameter. On the other hand, we would like to indicate particular behavior of the deceleration parameter q for $\alpha = 0.85$ and $\chi = 0.1$ indicating expansion with $q = 0$ for $z \in [1.1, 2.0]$.

2.1 Interacting model 1

There are various reasons to consider interacting DE models and some of them is cosmological coincidence problem. However, analysis presented in previous section demonstrated that suggested model is free from mentioned problem. On the other hand, study of the models involving interaction between dark components today is actual due to other reasons, for instance, relevant for structure formation study since interaction can leave unique impact in dynamics of universe and within observed structures shed the light on the physics of early universe. There are already well known forms of interactions considered in literature very intensively and one of them is given by Eq. (7). Consideration of it here, gives a model of universe, where EoS of DE reads as

$$\omega_{DE} = -\frac{3bH^2L + 2\sqrt{\Omega_{DE}}\dot{\hat{L}}_f + H\Omega_{DE}(\dot{\hat{L}}_f + 1)}{3H^2L_f\Omega_{DE}}. \quad (10)$$

While the form of the deceleration parameter

$$q = \frac{L((1-3b)H^2 + P_{DM}) - 2\sqrt{\Omega_{DE}}\dot{\hat{L}}_f}{2H^2L} -$$

$$-\frac{H\Omega_{DE}(\dot{\hat{L}}_f + 1)}{2H^2L}, \quad (11)$$

indicates phase transition between decelerated and accelerated expansions. Considered interaction will increase transition redshift z_{tr} and decrease present day value of the deceleration parameter with increasing interaction parameter b (for $\alpha = 1.15$, $\beta = 1.2$, $\alpha_0 = 0.15$, $\alpha_1 = 0.2$, $\alpha_2 = 0.25$, $H_0 = 0.7$ and $\Omega_{DM} = 0.27$). Study of this case shows that non-gravitational interaction for $\chi = 0.1$ will not change nature of DM, Eq. (4), while with $\chi = -0.1$ situation is different, namely, during evolution the sign of ω_{DM} will be changed indicating that at low redshift DM, Eq. (4), will evolve to DE. On the other hand, we found that non-gravitational interaction between considered dark components allows evolution of DE connecting early phantom phase (independently from sign of χ) with phantom phase of large scale universe. In both cases theoretical results are consistent with observational data.

2.2 Interacting model 2

To complete our study, we consider sign changeable interaction, Eq. (8), and found that such universe has the deceleration parameter with the graphical behavior presented in top panel of Fig. (2) indicating that increase of b will increase present day value of it for $\chi = 0.1$ and $\chi = -0.1$. On the other hand, increase of b will decrease z_{tr} for both cases. Bottom panel of Fig. (2) represents the graphical behavior of ω_{DE} and demonstrates that both cases ($\chi = 0.1$ and $\chi = -0.1$) can be supported by observational data, therefore, it is hard to conclude which one of considered models is favorable.

3 Om analysis and thermodynamics

Interesting aspect in study of accelerated expansion of low redshift universe and DE involves development of tools able to analyze models from different perspective. In this section we will concentrate our attention on Om analysis with

$$Om = \frac{x^2 - 1}{(1+z)^3 - 1}, \quad (12)$$

where $x = H/H_0$ and H_0 to be the value of the Hubble parameter at $z = 0$. It is assumed that if different trajectories have been obtained, then the

models are different. On the other hand, Om analysis has been generalized to three-point diagnostic $Om3$ and also to the two point Om

$$Om(z_2, z_1) = \frac{x(z_2)^2 - x(z_1)^2}{(1+z_2)^2 - (1+z_1)^2}. \quad (13)$$

Later, a small modification of two point Om has been suggested ($Om h^2$) in Ref. [35] and according to results for $z_1 = 0$, $z_2 = 0.57$ and $z_3 = 2.34$ reported in Ref. [35] the values for two point $Om h^2$ are

$$\begin{aligned} Om h^2(z_1; z_2) &= 0.124 \pm 0.045, \\ Om h^2(z_1; z_3) &= 0.122 \pm 0.01, \\ Om h^2(z_2; z_3) &= 0.122 \pm 0.012, \end{aligned} \quad (14)$$

while for Λ CDM the value is $Om h^2 = 0.1426$. Estimated values of $Om h^2$ for non - interacting model are presented in Table 1. It is evident, that with $\alpha = 0.95$ and $\chi = 0.1$ gives correct behavior for $Om h^2$. Moreover, already from studied behavior of cosmological parameters it has been seen, that $\chi = 0.1$ case is slightly favorable than $\chi = -0.1$ one (in scope of considered constraints on other parameters of the model). Combining obtained results we conclude, that for non - interacting model $\alpha \in (0.94, 1.18)$ should be considered in future for $\chi = 0.1$. Simple estimation of $Om h^2$ for other two cases indicates that, when interaction is given by Eq. (7) correct behavior for $Om h^2$ can be obtained with $\chi = -0.1$, $b = 0.03$ (even for higher values of b) and $\alpha = 0.15$. On the other hand, when interaction is given by Eq. (8), then again a narrow range around $\chi = -0.1$ will be favorable. To finalize study of suggested cosmological models question of validity of generalized second law of thermodynamics has been organized. Fig. (3) represents the graphical behavior of Om parameter and demonstrates validity of generalized second law of thermodynamics in absence of non - gravitational interaction. The validity of generalized second law of thermodynamics has been checked also for interacting models. The validity of generalized second law of thermodynamics has been studied taking into account that

$$\dot{S}_{tot} = \dot{S}_{de} + \dot{S}_{dm} + \dot{S}_h, \quad (15)$$

where $S_h = 8\pi^2 L^2$ it is the entropy associated with the horizon, while S_{dm} and S_{de} are the entropy associated with the dark matter and the dark energy, respectively, while dot represents the time derivative, which has been replaced by the derivative with

respect to redshift z . To obtain mentioned graphical behavior the following

$$TdS_i = dE_i + P_i dV, \quad (16)$$

with

$$E_i = \rho_i V, \quad (17)$$

for each component has been taken into account, where V is the volume of the system defined as follows

$$V = \frac{4\pi}{3} L^3. \quad (18)$$

4 Discussion

In this paper cosmological models are considered where two assumptions concerning to dark components are under consideration. In particular, it is assumed that one of dark components can be a model of generalized holographic DE with Nojiri - Odintsov cut - off of a specific form. The second dark component has been assumed to be viscous inhomogeneous fluid conditionally named DM. Moreover, two different type of non - gravitational interactions were taken into account having in mind that performed analysis will be used in future during study of structure formation. Cosmographic analysis of non - interacting model revealed that the model is free from cosmological coincidence problem, moreover, transition between decelerated and acceleration expansion phases exist. On the other hand, it has been observed, that decreasing of viscosity parameter χ will change nature of DE from phantom to quintessence (at higher redshifts), while at lower redshifts it is always a model of quintessence DE. Study of EoS of viscous fluid shows that it is usual fluid with $\omega_{DM} > 0$ at higher redshifts, but at lower redshifts it can evolve either into DE, or will continue to be usual fluid, for appropriate values of χ . Interesting feature of considered viscous fluid is transformation into pressureless fluid with ω_{DM} . Having this in mind, we consider two models involving two forms of interaction and observed that there are crucial aspects in dynamics of cosmological parameters differ observed for non - interacting model. In particular, we observed that in interacting models transitions are only between early time phantom and late time phantom universes. However, this helps also to have transition universe free from cosmological coincidence problem. It has been observed that increase of interaction parameter b will increase transition redshift, when interaction is

given by Eq. (7), while when interaction is given by Eq. (8), then transition redshift will decrease. In the last section of this work within the graphical behavior of the dynamics of entropy the validity of generalized second law has been demonstrated. Moreover, taking into account modified two point $Om h^2$ analysis and estimated values of it for $z_1 = 0$, $z_2 = 0.57$ and $z_3 = 2.34$, it has been found that for non - interacting model $\chi = 0.1$ is slightly favorable case then $\chi = -0.1$. Recall, that $\chi = 0.1$ provides DM model which with $\omega_{DM} > 0$ evolves to pressureless fluid with $\omega_{DM} = 0$. However, considered interactions support models with $\chi = -0.1$ providing DM to be a quintessence DE in recent universe. Discussed results are consistent with PLANCK 2015 constraints [36], however, for comprehensive picture a detailed study of the models within χ^2 should be organized. This we left to be discussed with structure formation study. On the other hand, due to the behavior of ω_{DE} at low redshifts interesting discussion can be organized involving study on future type singularities [37]. It is also possible to consider different forms of non - linear and non - linear sign changeable interactions involving different parameterizations of viscosity. Moreover, it will be interesting to study suggested model in case of existence of extra dimensions, since consideration of such situation provides modification of GR and solutions to problems of modern cosmology.

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О вязкой Вселенной голографической темной энергии с обрезками Ножири-Одинцова

В этой статье мы представляем результаты по изучению ускоренного расширения крупномасштабной Вселенной благодаря голографической темной энергии с ограничениями Нодзири-Одинцова, включая специфическую форму вязкости. Результаты были обсуждены в ходе конференций QFTG 2016 года и SCD 2016 года в Томске.

Keywords: ускоренное расширение, крупномасштабная Вселенная, темная энергия.

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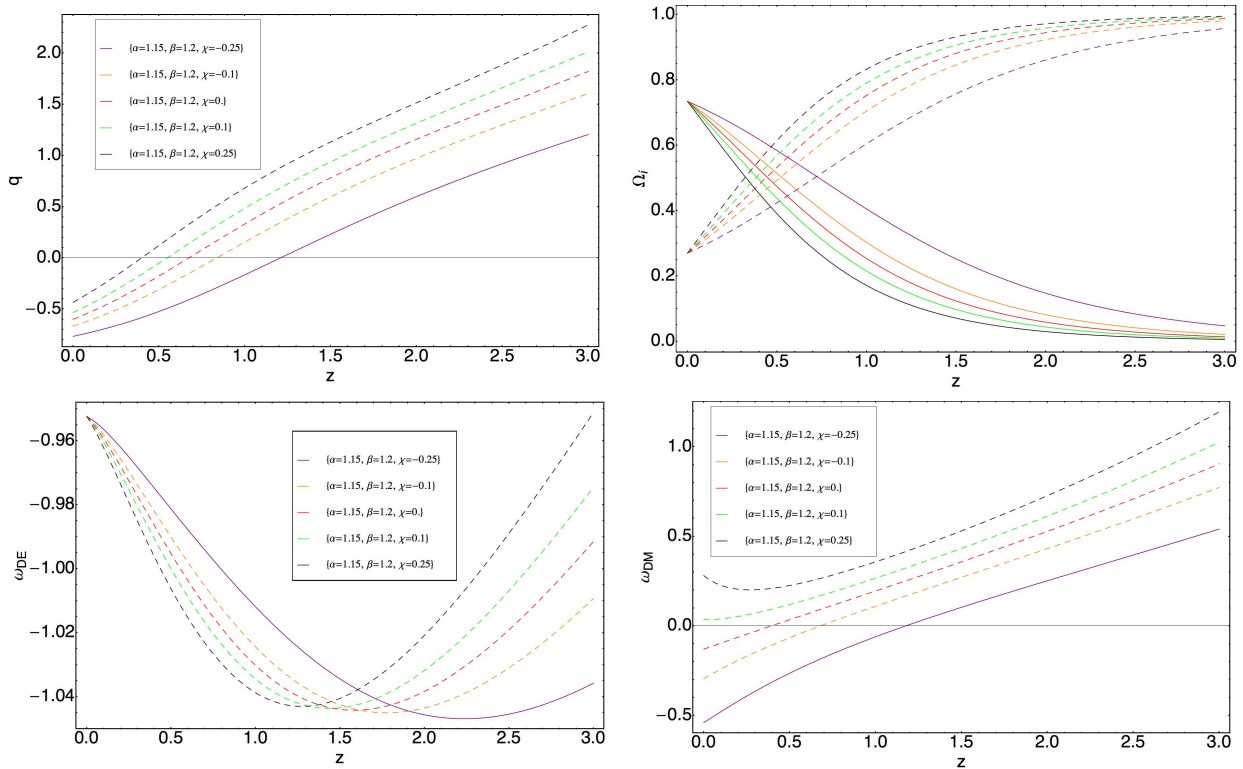


Figure 1: The graphical behavior of the deceleration parameter q and Ω_{DE} (solid lines) with Ω_{DM} (dashed lines) versus redshift z is presented by the top panel. The bottom panel represents the graphical behavior of ω_{DE} and ω_{DM} . Presented behavior for Ω_{DE} and Ω_{DM} is according to the same value of parameters as for q , ω_{DE} and ω_{DM} . Presented graphical behavior corresponds to non - interacting model.

(χ, α, b)	$Om h^2(z_1, z_2)$	$Om h^2(z_1, z_3)$	$Om h^2(z_2, z_3)$
$(-0.25, 1.15, 0)$	0.064	0.085	0.086
$(-0.1, 1.15, 0)$	0.099	0.174	0.181
$(0, 1.15, 0)$	0.126	0.255	0.266
$(0.1, 1.15, 0)$	0.153	0.355	0.372
$(0.25, 1.15, 0)$	0.196	0.549	0.579
$(-0.1, 0.75, 0)$	0.127	0.062	0.056
$(-0.1, 0.85, 0)$	0.122	0.073	0.069
$(-0.1, 0.95, 0)$	0.115	0.091	0.089
$(-0.1, 1.2, 0)$	0.096	0.222	0.233
$(0.1, 0.75, 0)$	0.167	0.082	0.075
$(0.1, 0.85, 0)$	0.165	0.102	0.097
$(0.1, 0.95, 0)$	0.162	0.136	0.133
$(0.1, 1.2, 0)$	0.151	0.547	0.582

Table 1: Estimated values of $Om h^2$ for non - interacting model

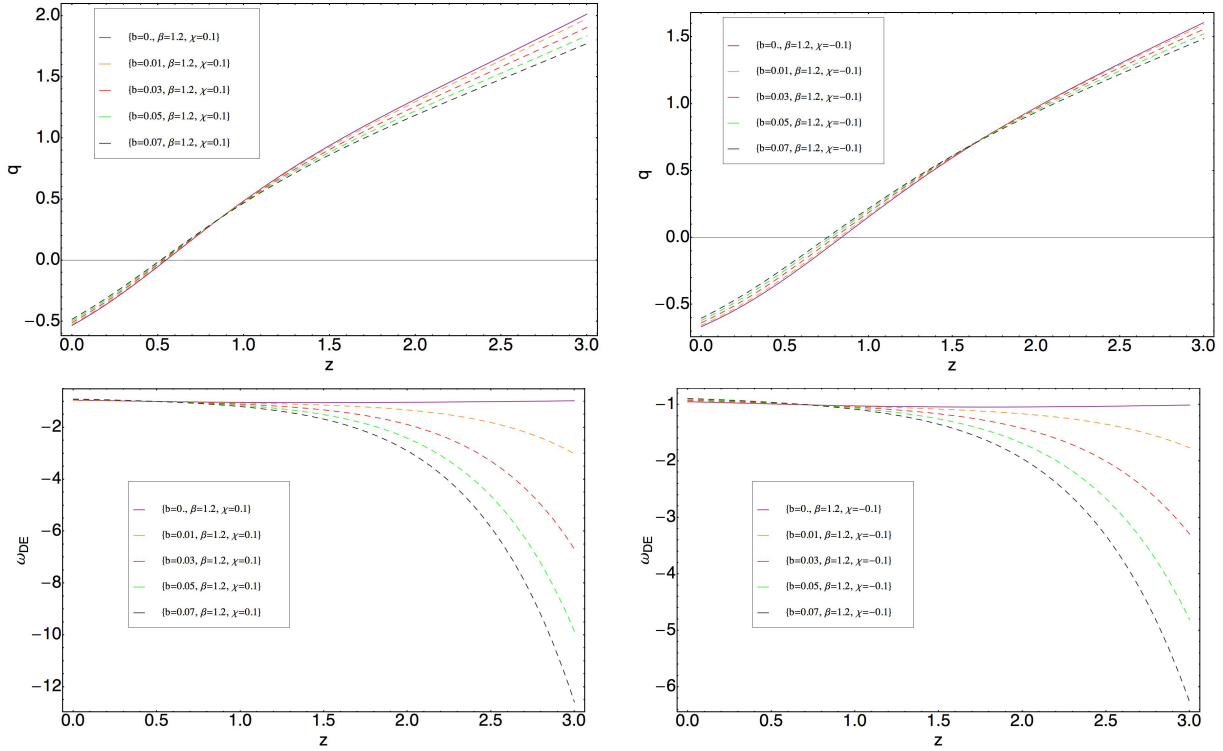


Figure 2: The graphical behavior of the deceleration parameter q and ω_{DE} versus redshift z corresponding to interacting model with interaction given by Eq. (8). The left panel corresponds $\chi = 0.1$ case, while the case corresponding $\chi = -0.1$ is presented on the right panel. $\alpha = 0.15$, $H_0 = 0.7$ and $\Omega_{DM} = 0.27$

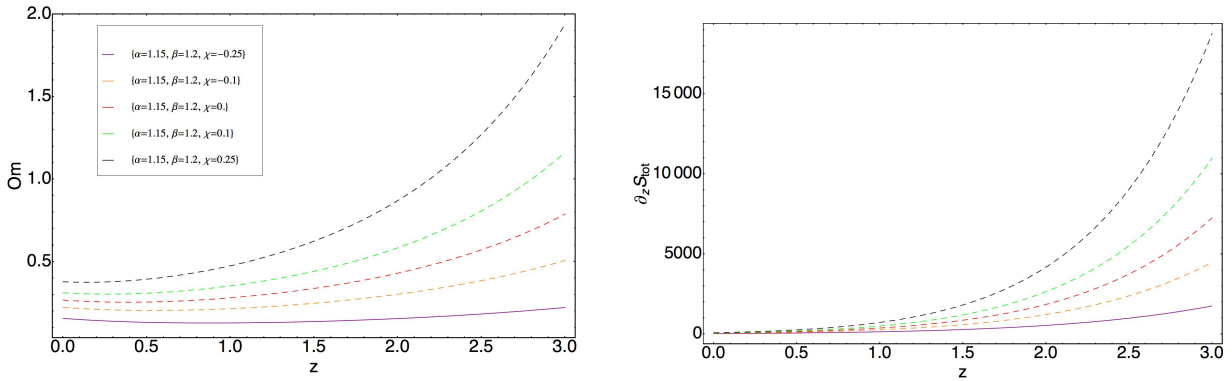


Figure 3: The graphical behavior of Om versus redshift z corresponds to the left plot. The right plot represents validity of generalized second law of thermodynamics. The case corresponds to non - interacting model

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