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Perfect fluid dark matter model revisited

We revisit certain features of an assumed spherically symmetric perfect fluid dark matter halo in the light of the observed data of our galaxy, the Milky Way (MW). The idea is to apply the Faber-Visser approach of combined observations of rotation curves and lensing to a first post-Newtonian approximation to «measure» the equation of state $\omega(r)$ of the perfect fluid galactic halo. However, for the model considered here, no constraints from lensing are used as it will be sufficient to consider only the rotation curve observations. The lensing mass together with other masses will be just computed using recent data.

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Dark matter is at the core of modern astrophysics. Many well known theoretical models for dark matter exist in the literature. In this paper, we shall revisit the model of perfect fluid dark matter, developed in Ref. [1], in the light of the observed/inferred data of our galaxy. The solution may be thought of as a dark matter induced spacetime embedded in a static cosmological Friedmann-Lemaitre-Robertson-Walker (FLRW) background. The model considered here assumes that a spherical dark matter distribution is the only gravitating source. Actually, there is practically little dark matter hidden in the disk. Hence, to explain the rotation curve measurements, we are forced to assume that dark matter in the halo region is spherically distributed and, if it is non-baryonic, would not be expected to collapse into a disk-like structure.

The general static spherically symmetric spacetime is represented by the following metric:

$$ds^2=-e^{\nu(r)}dt^2+e^{\lambda(r)}dr^2+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right), \quad (1)$$

where the functions $\nu(r)$ and $\lambda(r)$ are the metric potentials. For the perfect fluid, the matter energy momentum tensor T^{α}_{β} is given by $T^t_t=\rho(r)$, $T^r_r=T^{\theta}_{\theta}=T^{\varphi}_{\varphi}=p(r)$, where $\rho(r)$ is the energy density, $p(r)$ is the isotropic pressure. Considering flat rotation curve as an input, an exact solution of Einstein field equations is derived in [1]:

$$e^{\nu(r)}=B_0 r^l, \quad (2)$$

$$e^{\lambda(r)}=\frac{c}{a}+\frac{D}{r^a}, \quad (3)$$

$$a=-\frac{4(1+l)-l^2}{2+l}, \quad (4)$$

$$c=-\frac{4}{2+l}, \quad (5)$$

$$l=2v_c^2/c_0^2, \quad (6)$$

where $B_0>0$, D are integration constants and v_c is the circular velocity of stable circular hydrogen gas orbits treated as probe particles. The exact energy density and pressure are

$$\rho(r)=\frac{1}{8\pi}\left[\frac{l(4-l)}{4+4l-l^2}r^{-2}-\frac{D(6-l)(1+l)}{2+l}r^{\frac{l(2-l)}{2+l}}\right], \quad (7)$$

$$p(r)=\frac{1}{8\pi}\left[\frac{l^2}{4+4l-l^2}r^{-2}+D(1+l)r^{\frac{l(2-l)}{2+l}}\right]. \quad (8)$$

The free adjustable parameter D, having the dimension of (length)$^{-2}$, in the solution is extremely sensitive and its value can be decided only by observed physical constraints. In the present case, the constraint is that the galactic fluid be non-exotic and attractive, i.e., the equation of state parameter $\omega(r)=p(r)/\rho(r)\geq 0$ must hold within the halo radius.

In Ref. [2] Faber and Visser considered the metric in the form

$$ds^2=-e^{2\Phi(r)}dt^2+\frac{dr^2}{1-\frac{2m(r)}{r}}+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right). \quad (9)$$

Comparing it with the metric (1), we have

$$m(r)=\frac{r\left(1-c/a-D/r^a\right)}{2}, \quad (10)$$

$$\Phi(r)=\frac{\log B_0+l\log r}{2}. \quad (11)$$

The potentials $\Phi_{RC}(r)$ and $\Phi_{Lens}(r)$, obtained respectively from the rotation curve data and gravitational lensing observations, are derived to be

$$\Phi_{RC}(r)=\frac{\log B_0+l\log r}{2}, \quad (12)$$

$$\Phi_{Lens}(r)=\frac{\Phi(r)}{2}+\frac{1}{2}\int\frac{m(r)}{r^2}dr=\frac{\log B_0+l\log r}{4}+$$
$$+\frac{D(l+2)r^{\frac{4(1+l)-l^2}{2+l}}+l(l-4)\log r}{4\left(l^2-4l-4\right)}. \quad (13)$$

When the pressures and matter fluxes are small compared to the mass-energy density then $\Phi_{RC}(r)=\Phi_{Lens}(r)$, otherwise they may not be equal.

One pseudo-mass, inferred from rotation curve measurements, is given by

$$m_{RC}(r)=r^2\Phi'(r)=lr/2. \quad (14)$$

Another pseudo-mass, obtained from lensing measurements, is defined as

$$m_{Lens}(r)=\frac{r^2\Phi_{RC}(r)}{2}+\frac{m(r)}{2}=\frac{r\left[a\left(1+l-Dr^{-a}\right)-c\right]}{4a}. \quad (15)$$

For the equation of state parameter for perfect fluid, we should evaluate ω and impose the constraint that up to

$$\omega(r)=\frac{p_r(r)+2p_t(r)}{\rho(r)}\geq 0\,, \quad (16)$$

which will provide a limit on D. From the first order approximations of Einstein's equations, one obtains

$$\rho(r)=\frac{2m'_{Lens}(r)-m'_{RC}(r)}{4\pi r^2}=\frac{r^{(-2-a)}\left[-cr^a+a\left(r^a-D\right)+a^2D\right]}{8\pi a}, \quad (17)$$

$$p_r(r)+2p_t(r)=\frac{2\left[m'_{RC}(r)-m'_{Lens}(r)\right]}{4\pi r^2}=$$
$$=\frac{r^{(-2-a)}\left[cr^a-a^2D+a\left((l-1)r^a+D\right)\right]}{8\pi a}. \quad (18)$$

Then Eq. (16) yields

$$\omega(r)=\frac{p_r(r)+2p_t(r)}{3\rho(r)}\approx\frac{2}{3}\frac{m'_{RC}(r)-m'_{Lens}(r)}{2m'_{Lens}(r)-m'_{RC}(r)}=$$
$$=\frac{cr^a-a^2D+a\left[(l-1)r^a+D\right]}{3\left[-cr^a+a\left(r^a-D\right)+a^2D\right]}. \quad (19)$$

Observationally, such exact equalities as $p_r=p_t$ are impossible to attain. It follows that the difference in dimensionless pressures is not zero but [3]

$$4\pi r^2\left[p_r(r)-p_t(r)\right]=\frac{2}{r}\left(m_{RC}-m_{Lens}\right)-$$
$$-r\left[\frac{\left[m_{RC}-m_{Lens}\right]}{r}\right]'+O\left(\frac{2m}{r}\right)^2=$$
$$=\frac{r^{(-a)}\left[cr^a+2a^2D+a\left((l-1)r^a+D\right)\right]}{8\pi a}. \quad (20)$$

which is just the post-Newtonian version of isotropicity of the perfect fluid. However, this value of the right hand side for our galaxy is exceedingly small but not exactly zero.

The next issue is whether the model is Newonian or not, that is, how much of pressure contribution to mass is there. For this, we need to compare the Newtonian mass given by Eqs. (17) and (18),

$$M_N(r)=4\pi\int_0^r\rho(r)r^2dr=\frac{r\left(a-c-ar^{-a}D\right)}{2a} \quad (21)$$

and the mass in the first post-Newtonian approximation [2]

$$M_{pN}(r)=4\pi\int_0^r\left(\rho+p_r+2p_t\right)r^2dr=\frac{lr}{2}. \quad (22)$$

The Faber-Visser χ-factor, designed to provide a measure of the size of the pressure contribution, can be obtained from Eq.(19)

$$\chi(r)=\frac{m'_{Lens}(r)}{m'_{RC}(r)}=\frac{2+3\omega(r)}{2+6\omega(r)}. \quad (23)$$

There are recent works on constraining the mass and extent of the Milky Way's halo. We shall use a virial radius $R_{vir}\sim 200$ kpc, and a virial mass $M_{vir}\sim 1.5\times 10^{12}$ Solar masses [3]. We adopt them as the halo radius and mass of our galaxy.

Our strategy is to first find $\omega(r)$ from the Faber-Visser Eq. (19) using the input of v_c (that is, l) at some radius r. Next, within the halo boundary $R_{MW}\sim 200$ kpc, we impose the constraint $\omega(r<200\,kpc)>0$ which means attractive dark matter halo. At the boundary itself, we impose that $\omega(R_{MW})=0$ thereby allowing for a change of sign in $\omega(r)$ beyond the halo boundary. We then analyze in detail the numerical limits on $\omega(r)$ using the observed value of l and different signs of the adjustable parameter D.

Following Xue et al [4], we take $v_c(60kpc)=$ $=175$ km/s which means $l=2v_c^2/c_0^2=6.80\times 10^{-7}$. We now consider three cases of signs of parameter D.

The case $D=0$ imply that the perfect fluid approximates to dust dark matter (see Fig. 1). The case $D<0$ has a number of implications (Fig. 2). The last sector has positive energy density and negative pressure (Fig. 3), but the matter is not exotic as it still does not violate the Null Energy Condition (NEC). For $D>-4.84\times 10^{-18}$, the halo radius can be arbitrarily shifted away from 200 kpc (Fig. 4), which means that D can be adjusted to the possibility of having a larger Milky Way halo than considered here.

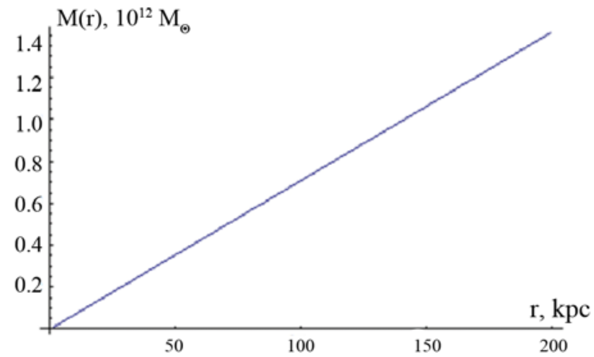

Fig. 1. Dust-like dark matter case

The case $D>0$ signals the presence of non-negligible pressure in the halo as opposed to the CDM paradigm but also leads to a singularity in $\omega(r)$ that can only be arbitrarily shifted at will by choosing D but not removed.

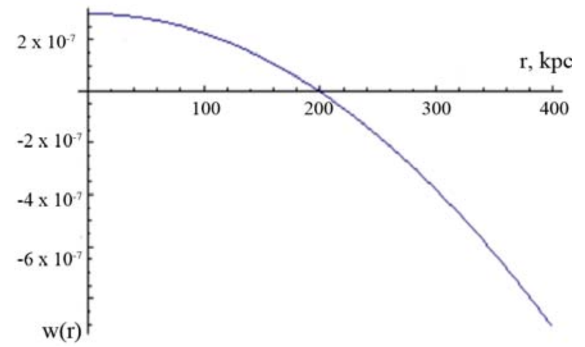

Fig. 2. The case $D < 0$

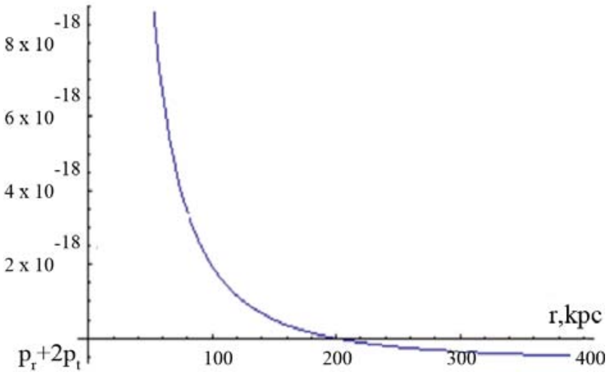

Fig. 3. Positive energy density and negative pressure region

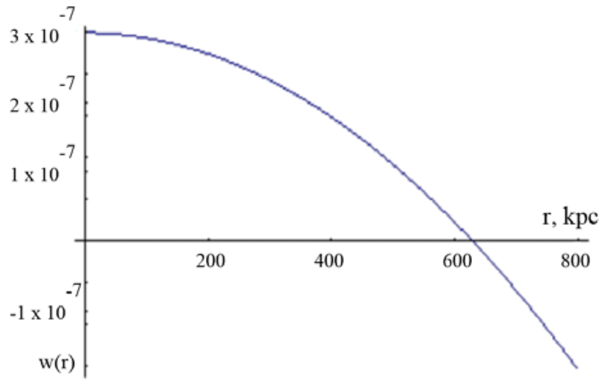

Fig. 4. The galactic halo radius

In view of the consistency with the recent galactic data we suggest an overall range $-4.84\times10^{-18} < D \leq 0$ which in turn leads to $0 \leq \omega(r) \leq 2.8\times10^{-7}$ for the perfect fluid singularity-free equation of state of dark matter. As we see, the values are concentrated around $D \sim 0$ leading to a strong constraint of dust-like dark matter which is supported also by CMB constraints [5, 6]. This is the main result of our paper.

References

1. Nandi K.K. Perfect fluid dark matter / K.K. Nandi, F. Rahaman, A. Bhadra, M. Kalam and K. Chakraborty // Physics Letters B. – 2010. – Vol. 694, Is. 1. – PP. 10–15.

2. Faber T. Combining rotation curves and gravitational lensing: how to measure the equation of state of dark matter in the galactic halo / T. Faber, M. Visser // Mon. Not. Roy. Astron. Soc. – 2006. – Vol. 372, Is. 1. – PP. 136–142.

3. Dehnen W. The velocity dispersion and mass profile of the Milky Way / W. Dehnen, D.E. McLaughlin and J. Sachania // Mon. Not. Roy. Astron. Soc. – 2006. – Vol. 369, Is. 4. – PP. 1688–1692.

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Пересмотр модели идеальной жидкости по отношению к темной материи

Пересматриваются некоторые особенности сферически симметричного гало Млечного пути, предположительно образованного темной материей, которая может быть моделирована как идеальная жидкость, на основе имеющихся наблюдательных данных. Ключевая идея состоит в том, чтобы применить формализм Фабера–Виссера, касающийся кривой вращения галактики и гравитационного линзирования, к первому постньютоновскому приближению, чтобы получить данные об уравнении состояния $\omega(r)$ идеальной жидкости, формирующей гало. Вообще говоря, для предполагаемой здесь модели нет жестких ограничений на основе эффекта линзирования – ограничения достаточным образом вводятся из данных, вытекающих из кривых вращения. Масса линзы-источника и другие характеристики вычислены с использованием последних данных.

Ключевые слова: темная материя, идеальная жидкость, уравнение состояния, массы галактик.