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Viscous fluid model in inflationary universe avoiding self-reproduction

We consider the description for the inflationary universe produced by viscous fluid in a flat Friedmann-Lemaitre-Robertson-Walker geometry. We have obtained the conditions of the existence regime of the inflation without self-reproduction for the theoretical inflationary model which agree with last results of the Planck satellite data.

Keywords: inflation, equation of state, viscosity, spectral index, tensor-to-scalar ratio.
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Recent observational data obtained in the BICEP2 experiment [1] and the Planck observational results for cosmic inflation [2, 3] lead to more detailed studying of inflation. Inflationary theory describes the very early intermediate stage of the evolution of the Universe. The stage of inflation can be extremely short, but the universe within this time becomes exponentially large.

However, the inflationary theory has the problems of multiversity, predictability and initial conditions. The inflationary scenario which avoids self-reproduction and resolves these problems has been proposed in [4]. There are several different regimes which are possible in inflationary cosmology. We consider the simplest regime that is inflation without self-reproduction. In this scenario the universe is not stationary. Self-reproduction of the inflationary universe does not allow inflation to finish, it would never end.

It is shown in [5], $F(R)$ inflation without self-reproduction may be formulated analogously with the corresponding scalar models considered in [4]. The description of inflationary universe from perfect fluid and $F(R)$ gravity and its comparison with observational data was considered in [6]. A general review of inflationary cosmology and resolution problems of multiversity and initial conditions is given in [7].

In order to avoid the self-reproduction at the inflationary universe the thermodynamic parameter in the equation of state must simultaneously satisfy the following requirements [4]:

$1+\omega(N)=1$ at $N=1$ (to have graceful exit),

$$1+\omega(N)\le\frac{2}{3} \text{ at } N=N_m \quad (1)$$

(to solve initial condition problem); $1+\omega(N)<<1$ for $1<N<N_m$ (inflation); $1+\omega(N)>\varepsilon(N)$ for $1<N<N_m$ (no self-reproduction).

Further, we will consider the description of inflation with viscosity in terms of inhomogeneous equation of state with e-folding parameter N, which is defined through the scale factor a as $a=a_0e^{-N}$, where a_0 is a constant and $\omega(N)$ is the equation of state parameter. The number e-folds play a role of time. The energy density of the Planck unit $\varepsilon(N)$ is connected with energy density of the usual unit by relation $\varepsilon(N)=k^4\rho(N)$.

The inhomogeneous fluids in terms time-dependent equation of state in the presence of viscosity were considered in papers [9–12]. Inhomogeneous fluid cosmology may be interpreted as modified gravity [13, 14]. Various examples inhomogeneous viscous coupled fluids were investigated in [15–17]. Paper [18] was devoted to inflationary cosmological models with viscous coupled fluids.

In this article we will reproduce the inflation by using an inhomogeneous equation of state parameter and formulate the conditions for existence of inflation in the regime of no self-reproduction for the viscous fluid models satisfying the Planck and BICEP2 results.

Inflationary model with viscosity avoiding self-reproduction

We will investigate the conditions in the early-time universe which allow avoiding the regime of self-reproduction. These conditions are described in terms of the equation of state parameter and the bulk viscosity. Note that we consider the inflation in absence of the matter. Further, we apply the formalism of inhomogeneous viscous fluid in a flat Freedman-Lemaitre-Robertson-Walker (FLRM) space-time.

The gravitational field equations for a perfect fluid in the FLRM space-time have the following form:

$$\rho'(N)+3\left[P(N)+\rho(N)\right]=0,$$
$$-\frac{2}{k^2}H(N)H'(N)=\rho+P. \quad (2)$$

Here the Hubble parameter is defined as $H\equiv\frac{\dot{a}}{a}$, where $a(t)$ is the scale factor and $k^2=8\pi G$ with G being Newton's gravitational constant. The dot denotes the derivative with respect to time t. P and ρ are the pressure and energy density of the perfect fluid respectively. Here the prime operating $\rho'(N)$ and $H'(N)$ means the derivative with respect to e-folding parameter N, that is $\rho'(N)\equiv\frac{d\rho(N)}{dN}$ and $H'(N)\equiv\frac{dH(N)}{dN}$.

We now write the Friedmann's equation for the Hubble parameter:

$$\frac{3}{k^2}H^2(N)=\rho. \quad (3)$$

We take the equation of state to have the inhomogeneous form:

$$P(N)=\omega(N)\rho(N)+\zeta(N), \tag{4}$$

where $\zeta(N)$ is the viscosity.

Further we will study the conditions (1) for the inflationary model with viscosity on the subject avoiding the self-reproduction.

Let us consider the following linear form of the Hubble parameter [6]:

$$H(N)=G_0 N+G_1, \tag{5}$$

where $G_0<0$ and $G_1>0$ are the parameters.

We choose viscosity to be proportional to the square of the parameter H [18]:

$$\zeta(N)=\theta H^2(N), \tag{6}$$

where θ is positive dimensional constant. In geometric units, its dimension is equal $[\theta]=cm^{-2}$.

Using the gravitational equation of motion (2) with (5) and (6), we obtain the expression for the thermodynamic parameter:

$$\omega(N)=-1-\frac{1}{3}\left[\frac{2G_0}{H(N)}+k^2\theta\right]. \tag{7}$$

The second term in the square bracket represents the contribution of the viscosity.

Therefore, the equation of state (4) reads:

$$p(N)=\left[-1-\frac{2G_0}{H(N)}\right]\rho(N). \tag{8}$$

Now we define conditions, when the thermodynamic parameter (7) in the equation of state satisfies simultaneously all requirements of avoiding self-reproduction (1). Checking condition (1a), we obtain:

$$\theta=\frac{1}{k^2}\left(\frac{2|G_0|}{G_0+G_1}-3\right). \tag{9}$$

Here $|G_0|<G_1<\frac{5}{3}|G_0|$, because $\theta>0$. From the condition (1b) we become the meaning of e-folding parameter N at the beginning of inflation:

$$N_m=\frac{G_1}{|G_0|}-\frac{2}{2+k^2\theta}. \tag{10}$$

Thus, initial condition problem is solved at $N\simeq N_m$.

Inflation take place if condition (1c) is fulfilled, that is

$$1<N<\frac{G_1}{|G_0|}-\frac{2}{3+k^2\theta}=N'_m. \tag{11}$$

Consequently, the conditions (1b–1c) can simultaneously be satisfied if we assume that $1<N<N_m$ $(N_m<N'_m)$. Now we consider the condition (1d) «no self-reproduction» of inflation. Taken into account (7), this condition can be simplified as:

$$9H^3(N)+\theta H(N)+\frac{2G_0}{k^2}<0. \tag{12}$$

The inequality (12) has the solution:

$$1<N<\frac{1}{|G_0|}\left(G_1-\frac{2}{3}\sqrt{\frac{\theta}{3}}\operatorname{ctg}2\alpha\right), \tag{13}$$

where $\operatorname{tg}\alpha=\sqrt[3]{tg\left[\frac{1}{2}\operatorname{arctg}\frac{k^2}{3|G_0|}\left(\frac{\theta}{3}\right)^{\frac{3}{2}}\right]}$, $|\alpha|\le\frac{\pi}{4}$. From the comparison (10) and (13) in order to avoid initial condition problem it is necessary that took place the following dependence between the parameters:

$$G_0+G_1=\frac{2}{3}\sqrt{\frac{\theta}{3}}\operatorname{ctg}2\alpha. \tag{14}$$

Thus, we have obtained for the inflationary model with linear form for H the equation of state in the form (8). If we want to avoid the self-reproduction and the problem of initial conditions, the thermodynamic parameter (7) in the equation of state must simultaneously satisfy the requirements (9), (10) and (14).

Comparison of inflationary model with observational data

In this section we will calculate the inflationary parameters and consider the correspondence between the spectral index and tensor-to-scalar ratio with Planck and BICEP2 data.

At first calculate the so called «slow-roll» slope parameter [8]:

$$\varepsilon=-\frac{\dot{H}}{H^2}. \tag{15}$$

For the linear form of the Hubble parameter (6) it is equal:

$$\varepsilon=\frac{|G_0|}{G_0 N+G_1}. \tag{16}$$

In order to have acceleration, one must require $\varepsilon<1$. In our case that corresponds:

$$1<N<\frac{G_1-|G_0|}{|G_0|}. \tag{17}$$

Another important «slow-roll» parameter in studying inflation is calculated by [8]:

$$\eta=\varepsilon-\frac{1}{2\varepsilon H}\dot{\varepsilon}. \tag{18}$$

I our case it take place $\eta=\frac{\varepsilon}{2}$. The power spectrum is [8]:

$$\Delta_R^2=\frac{k^2H^2}{8\pi^2\varepsilon}. \tag{19}$$

With equations (5) and (36) we find

$$\Delta_R^2=\frac{k^2(G_0N+G_1)^3}{8\pi|G_0|}. \tag{20}$$

Taking into account the «slow-roll» parameters, one can calculate the spectral index n_s and the tensor-to-scalar ratio r:

$$n_s=1-6\varepsilon+2\eta, \quad r=16\varepsilon. \tag{21}$$

We obtain

$$n_s = 1 - \frac{5|G_0|}{G_0 N + G_1}, \quad r = \frac{16|G_0|}{G_0 N + G_1}. \qquad (22)$$

From the observation results by the Planck satellite, it is obtained that $n_s = 0.9603 \pm 0.0073$. The satisfaction with this result can be reach if we require that $\frac{|G_0|}{G_0 N + G_1} = 0.00794 \pm 0.000146$.

The relation between the equation of state parameter and tensor-to-scalar ratio has a view:

$$\omega(N) = -1 + \frac{r}{24} - \frac{1}{3} k^2 \theta. \qquad (23)$$

Let us consider the generalization of the linear form for Hubble parameter:

$$H(N) = G_0 N^n + G_1, \qquad (24)$$

where the parameter n is positive integer, $G_0 < 0$ and $G_1 > 0$.

We will check correspondence this theoretical model with Planck observational data. At first we calculate the slow-roll parameters:

$$\varepsilon = \frac{n|G_0|N^{n-1}}{H}, \quad \eta = \frac{1}{2}\left(\varepsilon - \frac{n-1}{N}\right). \qquad (25)$$

It take place the regime of the universe acceleration, if fulfills the inequality $\varepsilon < 1$. In our case it is equivalent the following condition:

$$N^n + nN^{n-1} - \frac{G_1}{|G_0|} < 0. \qquad (26)$$

For example, at $n = 2$ we obtain the solution:

$$1 < N < -1 + \sqrt{1 + \frac{G_1}{|G_0|}}. \qquad (27)$$

Let us consider the asymptotic case, when $N \gg 1$. Then, the inequality (26) simplifies and by keeping only order term we obtain:

$$N^n < \frac{G_1}{|G_0|}. \qquad (28)$$

From here the e-folding parameter N changes in the region $1 << N < \left(\frac{G_1}{|G_0|}\right)^{\frac{1}{n}}$.

Now, we find the power spectrum:

$$\Delta_R^2 = \frac{k^2 \left(G_0 N^n + G_1\right)^3}{8\pi n |G_0| N^{n-1}}. \qquad (29)$$

The spectral index n_s and the tensor-to-scalar ratio r are equal:

$$n_s = 1 - 5\varepsilon - \frac{n-1}{N}, \quad r = \frac{16n|G_0|N^{n-1}}{G_0 N^n + G_1}. \qquad (30)$$

Further, we discuss the coincidence of this inflationary model with Planck observation results. For the reproducing of the observations it is necessary to demand:

$$\frac{(4n+1)|G_0|N^n + (n-1)G_1}{N\left(G_0 N^n + G_1\right)} = 0.0397 \pm 0.0073. \qquad (31)$$

Another restriction looks as:

$$\frac{n|G_0|N^{n-1}}{G_0 N^n + G_1} < 0,006875. \qquad (32)$$

Thus, we conclude the capacity of this model to describe the evolution of the inflationary universe.

Conclusion

In this paper we have considered the description of inflation in perfect fluid model taking into account the viscosity properties in a Friedmann-Lemaitre-Robertson-Walker flat space-time. We paid attention to conditions, which are necessary to avoid the self-reproduction of the very-early universe in the inflationary epoch. For this purpose the expressions for thermodynamic parameter in the equation of state for three inflationary models in terms of the number of e-folds are obtained. We have analyzed in every model the conditions of "no self-reproduction" proposed in paper [4]. It is demonstrated that perfect fluid description may lead to such inflationary universe. The expressions for spectral index, the tensor-to-scalar ratio and power spectrum are calculated. Also we have expressed the thermodynamic parameter via tensor-to-scalar ratio and the viscosity. The detailed consideration of this problem one can find in [19].

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Тимошкин А.В.

Модель вязкой жидкости в инфляционной Вселенной без самовоспроизведения

Рассматривается описание инфляционной Вселенной, индуцированной вязкой жидкостью в плоской геометрии Фридмана–Леметра–Робертсона–Уокера. Получены условия существования режима инфляции без самовоспроизведения в теоретической модели инфляции, которая согласуется с последними данными спутников Планка.

Ключевые слова: инфляция, уравнение состояния, вязкость, спектральный индекс, скалярно-тензорное отношение.